General information

The exam will be an oral exam lasting twenty minutes. The schedule will be posted on the course website.

At least fifty percent of the exam questions will be taken from the list below. This list is intended to facilitate your review process and to give an idea of the types of questions I will ask. However, this list is not necessarily exhaustive: any material which was covered in class and which was not explicitly announced as being non-examinable is fair game.

Basic symplectic geometry

- Give five examples of symplectic manifolds. (Each of which should be distinct in a non-silly way, i.e. you can't take $(\mathbb{R}^{2n}, \omega_0)$ for $n = 1, 2 \ldots, 5$.)
- Which spheres admit a symplectic structure?
- What is the statement of the Darboux theorem?
- What is a Hamiltonian? Prove that the time-1 flow of a (possibly time-dependent) Hamiltonian is a symplectic isomorphism (also called symplectomorphism).
- Let (M, ω) be a connected symplectic manifold. Prove that the group of symplectic isomorphisms acts transitively on points (i.e. you can send any point to any other point).
- What is a symplectic isotopy? What is a Hamiltonian isotopy? Give an example of a symplectic isotopy which is not Hamiltonian.
- Sketch the proof that any two smooth complex hypersurfaces in $\mathbb{CP}^n$ of the same degree are symplectomorphic.

J-holomorphic curves

- What is an almost-complex structure? What does it mean for an almost-complex structure to be compatible with a symplectic form? Given a symplectic manifold (M, ω), what can you say about the topology of the space of compatible almost-complex structures?
- What is a J-holomorphic curve?
- What is the energy of a J-holomorphic curve?

- Let (M, ω) be closed symplectic and let J be a compatible almost-complex structure. If Σ is a compact Riemann surface (possibly with boundary) and $u : \Sigma \to M$ is J-holomorphic, prove that $\int_\Sigma u^*\omega \geq 0$ with equality iff $u \equiv 0$.
- Suppose that J is a compatible almost-complex structure on $(\mathbb{R}^{2n}, \omega_0)$. Prove that there does not exist a nonconstant closed J-holomorphic curve.
- Let (M, ω) be closed symplectic and let J be a compatible almost-complex structure. Suppose that $u_k : \mathbb{CP}^1 \to M$ is a sequence of J-holomorphic curves, and suppose that there exists $C > 0$ such that $\int_{\mathbb{CP}^1} u_k^*\omega < C$ for all k. Prove or give a counterexample: the u_k represent finitely many distinct classes in $H_2(M; \mathbb{Z})$. *Hint: Gromov compactness*
- What is the Novikov ring/field?
- What is quantum cohomology? Why is quantum cohomology typically only well-defined over the Novikov ring? Why is it typically only $\mathbb{Z}/2$-graded? Why can the $\mathbb{Z}/2$-grading be lifted to a $\mathbb{Z}$-grading under the assumption that the first Chern class is 2-torsion?
- Tell me about the quantum cohomology of $\mathbb{CP}^n$? of the n-torus? (Hint: see the optional exercise sheets)

Morse theory and Hamiltonian Floer theory

- What is a Morse function? Do they always exist?
- Sketch the definition of Morse (co)homology. Why does the differential square to zero?
- What does it mean for a (possibly-time dependent) Hamiltonian to be non-degenerate?
- Tell me about Arnold's conjecture? How is Floer (co)homology relevant to this?
- Assuming I already know about Morse (co)homology, sketch the construction of Hamiltonian Floer (co)homology for a non-degenerate Hamiltonian, in the simplest setting. What are the generators of the complex, how to define the differential? Why does the differential square to zero?

- What is sphere bubbling? Why does it obstruct the differential squaring to zero in Hamiltonian Floer theory? What assumptions can you impose on a symplectic manifold to prevent it?
- If (M, ω) is symplectically aspherical (meaning that ω acts by zero on $\pi_2(M)$), why does this allow us to define Hamiltonian Floer (co)homology with $\mathbb{Z}/2$-coefficients? Why must we use Novikov coefficients in general?

Lagrangian Floer theory and Fukaya categories

- What are your favorite Lagrangian submanifolds?
- What is an exact symplectic manifold? what is an exact Lagrangian submanifold?
- What is disk bubbling? Why is it a problem for defining Lagrangian Floer cohomology? Why does this problem not occur if the ambient symplectic manifold is exact and all Lagrangians under consideration are exact?
- Prove that there does not exist a closed exact Lagrangian in $(\mathbb{R}^{2n}, \frac{1}{2}d(\sum_i x_i dy_i - y_i dx_i))$. *Hint: Argue that any compact subset of* $\mathbb{R}^{2n}$ *can be displaced from itself by a Hamiltonian isotopy. Review the main properties of Floer cohomology for exact Lagrangians.*
- What is the statement of the Nearby Lagrangian Conjecture? What is known about it?
- What is a Liouville manifold?
- Explain why $(\mathbb{R}^2, xdy - ydx)$ a Liouville manifold. Draw the associated Liouville vector field. Same question for T^*Q, for Q a closed manifold.
- Sketch the definition of a category. What does it mean for a category to be k-linear, for k a commutative ring?
- What is an equivalence of categories? What is the difference between equivalence and isomorphism of categories?
- Sketch the definition of a triangulated category (roughly). Examples?
- What is a dg category? Examples? What is an equivalence of dg categories?

- What is a dg enhancement of a triangulated category?
- Why does the bounded derived category of an abelian category admit a canonical dg enhancement?
- What is a Morita-invariant property?
- Let $\mathcal{F}uk(-)$ be the Fukaya category of closed exact Lagrangians (with $\mathbb{Z}/2$-coefficients and $\mathbb{Z}/2$-grading). Let Q be a closed manifold. What is $\mathrm{Perf}(\mathcal{F}uk(T^*Q))$? (You don't need to prove anything since I also didn't prove this in class, but you should know the answer). Same question for the Fukaya category of $\widetilde{\mathbb{C}^2/\Gamma}$, for $\Gamma < SU(2)$ a finite subgroup.
- What is the statement of homological mirror symmetry for Calabi–Yau hypersurfaces in complex projective space?