User Defined by Curvature*

A planar curve (parametrized by arc length) can be reconstructed from its curvature function $t \mapsto \kappa(t)$ as follows:

(1) take the antiderivative of κ, $\alpha(t) := \int^t \kappa(\sigma) d\sigma$,

(2) choose an initial point p, an initial tangent vector $\dot{c}(0)$ and an orthonormal basis $e_1 = \dot{c}(0),\ e_2$,

so the definition of curvature (namely $\kappa := |\ddot{c}|$, plus a sign convention) implies that,

(3) $\dot{c}(t) = e_1 \cdot \cos\alpha(t) + e_2 \cdot \sin\alpha(t)$.

Then one more integration,

(4) $c(t) = p + \int_0^t \dot{c}(\sigma) d\sigma$,

determines the curve. This description explains why the curvature is also called the "rotation speed" of the tangent vector field $\dot{c}(t)$.

In 3D-XplorMath one can select *User Curvature*. A dialog box opens and one can enter the desired curvature function. The initial point p is taken as the origin and the initial tangent is taken as the unit vector in the positive x-direction.

The parameter gg in this case defines a "precision divisor", that can be between 1 and 30. The size of the

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* This file is from the 3D-XplorMath project. Please see: http://3D-XplorMath.org/

subintervals used in approximating the above integrals is $\delta := (tMax - tMin)/(tResolution - 1)$ if $gg = 1$, and in general it is δ/gg. If the curvature function κ becomes very large somewhere, and in particular if it is infinite at an endpoint of the interval $[tMin, tMax]$, it is a good idea to use a fairly large value of gg to counteract the resulting numerical inaccuracies that will occur in the evaluation of the integrals.

Note that 3D-XplorMath offers the same Action Menu Entries as for explicitly parametrized curves. For example try the caustics.

R.S.P.