Third exercise sheet Advanced Algebra II.

Problem 1 (4 points). *Give an example showing that the category of sheaves of abelian groups on a topological space X can fail to be AB4*!*

Problem 2 (3 points). *Let $U \xrightarrow{j} X$ be the inclusion of an open subset $U \subseteq X$ to a topological space X, and let $\mathcal{F}$ be a sheaf of abelian groups on U. For open $V \subseteq X$, let $j_!\mathcal{F}(V)$ be the set of all $f \in \mathcal{F}(U \cap V)$ for which there is an open neighbourhood W of $V \setminus U$ in V such that $f|_{W\cap U} = 0$. Show that $j_!\mathcal{F}$ is a sheaf of abelian groups on X, and calculate its stalks!*

Problem 3 (6 points). *If j is as in the previous exercise, construct a pair of adjoint functors between the categories of sheaves of abelian groups on U and on X in which $j_!$ is the left and the functor j^* of restriction to U the right adjoint functor!*

Problem 4 (1 points). *In the situation of the previous two problems, show that j^* preserves injectivity of sheaves!*

Problem 5 (1 point). *For an arbitrary topological space X, show that the category of sheaves of abelian groups on X has a generator!*

Problem 6 (5 points). *Let X be a spectral space. Show that X is Hausdorff if and only if for every spectral space T, every continuous map $T \xrightarrow{t} X$ is spectral!*

Solutions should be submitted in the lecture Friday, May 3.