Minisymposium 12

Representation Theory of Algebras

Leiter des Symposiums:

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Beringstraße 1
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Donnerstag, 21. September
Hörsaal 118, AVZ I, Endenicher Allee 11-13

15:00 – 15:50 **Idun Reiten** *(Trondheim)*
Relative Calabi-Yau duality

16:00 – 16:50 **Bernard Leclerc** *(Caen)*
Cluster algebra structures on coordinate rings of partial flag varieties

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17:00 – 17:50 **Markus Reineke** *(Münster)*
Smooth models of quiver moduli

Mittwoch, 20. September
Hörsaal 118, AVZ I, Endenicher Allee 11-13

15:00 – 15:50 **Christof Geiss** *(UNAM, Mexico)*
Examples of higher Auslander algebras which are quasi-hereditary

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16:00 – 16:50 **Henning Krause** *(Paderborn)*
Adams resolutions for modular representations

17:00 – 17:50 **Claus Michael Ringel** *(Bielefeld)*
Take-off subcategories

Vortragsauszüge

Idun Reiten *(Trondheim)*
Relative Calabi-Yau duality

Triangulated categories of Calabi-Yau dimension 2 have turned out to be important in connection with cluster algebras, since for example the cluster categories and the stable categories of preprojective algebras of Dynkin diagrams have this property. In these examples, the triangulated categories contain cluster-tilting objects (i.e. maximal exceptional objects) and are closely related to the module category of the endomorphism algebra of each cluster-tilting object. We call such endomorphism algebras 2-Calabi-Yau tilted (and cluster-tilted if they arise from cluster categories). We show that they are Gorenstein of dimension at most one and that their stable categories of Cohen-Macaulay modules are Calabi-Yau of dimension 3. Often, a 2-Calabi-Yau tilted algebra naturally arises as a quotient of an algebra of finite global dimension. This algebra then satisfies a relative 3-Calabi-Yau property, as first shown by Geiss-Leclerc-Schroer in the context of modules over preprojective algebras. In contrast to its “absolute” variant, this relative Calabi-Yau property generalizes to higher dimensions.

Bernard Leclerc *(Caen)*
Cluster algebra structures on coordinate rings of partial flag varieties

This is a joint work with Christof Geiss and Jan Schröer. We investigate subcategories of the category of modules over a preprojective algebra Λ of Dynkin type (A,D,E) of the form Sub Q, where Q is an injective Λ-module. In particular, we construct explicit maximal rigid modules in Sub Q and define a mutation operation between maximal rigid modules. This is then applied to introduce a cluster algebra structure on the homogeneous coordinate rings of the generalized flag varieties G/P where G is the complex semisimple simply connected algebraic group with the same Dynkin type as Λ, and P is a parabolic subgroup.

Markus Reineke *(Münster)*
Smooth models of quiver moduli

Quiver moduli parametrize isomorphism classes of (poly-)stable representations of quivers up to isomorphism. Analogous to the case of moduli of vector bundles, there is a distinction between a (numerically defined) coprime case, with quite well-understood non-singular projective moduli, and a non-coprime case, leading either to non-compact, or to highly singular moduli.
The aim of the talk is to formulate and study a closely related moduli problem, which always produces smooth projective moduli. Their topology and geometry (in particular, their Betti numbers) will be described, and the representation-theoretic significance will be discussed.

Christof Geiss *(UNAM, Mexico)*
Examples of higher Auslander algebras which are quasi-hereditary

This is a report on joint work in progress with B. Leclerc (Caen) and J. Schröer (Bonn). Let Q be a Dynkin quiver, $\bar{Q}$ its double and $\Lambda = k\bar{Q}/(\sum_{a\in Q_1}[a,\bar{a}])$ the corresponding preprojective algebra. Let R be a maximal 1-orthogonal Λ-module and $E = \mathrm{End}_\Lambda(R)$, so this is a higher Auslander algebra in the sense of Iyama. Then $F_R = \mathrm{Hom}_\Lambda(-,R)$ induces an anti-equivalence from Λ-modules to the E-modules of projective dimension at most 1. If R is produced by pushing the projective modules of the (ordinary) Auslander algebra of kQ to Λ then E is canonically quasi-hereditary. The image of F_R are precisely the Δ-good modules, the Δ_i are just $F_R(X)$ where X runs over the indecomposable kQ^{op}-modules viewed as Λ-modules, and $\mathrm{Ext}^1_\Lambda(F_R(X),F_R(Y)) \cong D\,\mathrm{Ext}^1_{Q^{\mathrm{op}}}(X,Y)$.
The Δ-dimension vectors should in this way provide usefull invariants of Λ-modules.

Henning Krause *(Paderborn)*
Adams resolutions for modular representations

For any modular representation (of some finite group) an Adams resolution is constructed. This resolution helps to determine the cohomological support. The talk presents recent joint work with Dave Benson and Srikanth Iyengar.

Claus Michael Ringel *(Bielefeld)*

Take-off subcategories

Let Λ be an artin algebra and mod Λ the category of left Λ-modules of finite length. A full subcategory of mod Λ will be said to be a *take-off subcategory* provided it is closed under cogeneration, contains infinitely many isomorphism classes of indecomposable modules, and is minimal with these properties. We show the existence of take-off subcategories (provided, of course, that Λ is representation-infinite).