Minisymposium 8 - Homogenisierung und Anwendungen

Christoph Schwab (ETH Zürich)

NUMERICAL SOLUTION OF ELLIPTIC PROBLEMS WITH STOCHASTIC COEFFICIENTS

A scalar, elliptic boundary value problem in divergence form with stochastic diffusion coefficient $a(x,\omega)$ in a bounded domain $D \subset \mathbb{R}^d$ is reformulated as a deterministic, infinite-dimensional, parametric problem by Karhunen-Loève or Legendre expansions of $a(x,\omega)$.
Deterministic solvers based on a) mean square projection of this problem into a product probability space of finite dimension M and b) sparse Galerkin discretizations of the M dimensional parametric problem of wavelet resp. spectral (or polynomial chaos in the sense of N. Wiener) type.
The convergence rate of the resulting deterministic solution algorithm is analyzed in terms of the dimension M and in terms of the number N of deterministic problems to be solved as both, M and the resolution level of the wavelet discretization resp. the degree of the polynomial expansion increase simultaneously.
New analytic regularity estimates of the solution of the truncated parametric deterministic problems which are explicit the truncation dimension M are given. Optimal convergence rates of the sparse Galerkin approximations of the random solution, in terms of the number N of deterministic problems to be solved, are proved when the dimension M of the parameter space increases simultaneously with the meshwidth in the sparse wavelet approximation resp. the spectral order in the chaos approximation are increased.