INTRODUCTION TO THE MINIMAL MODEL PROGRAM

Abstract

The Minimal Model Program (MMP) is the branch of algebraic geometry which deals with the classification of projective varieties up to birational equivalence. More precisely, given a smooth variety X, the MMP aims to construct a birational model $X \rightarrow X_{min}$ - the minimal model of X - which is "as simple as possible". Albeit being a very classic problem in algebraic geometry, it took more than 50 years, and a considerable amount of work, just to understand how to coherently generalise the surface case to higher dimensions.

The goal of the talk is to give a gentle introduction to the subject, with as few technicalities as possible. I will begin by recalling some basic facts and constructions in algebraic geometry, then move on to the definition of minimal surface, as proposed by the Italian school of algebraic geometry in the beginning of 20th century. The second part of the talk is about MMP in dimension ≥ 3 . Here there will be some interesting new features with respect to the case of surfaces, namely small contractions and flips, which will inevitably led us to consider also singular varieties in our treatment. We will discuss some major theorems and conjectures (e.g. Abundance and Termination of Flips) and present the full MMP in dimension three. In the end we will motivate the generalisation of MMP to the logarithmic category.

Duration: Approx 90 minutes. I will try to recall as many prerequisites as possible in the first part of the talk. The audience should be comfortable with the notions of complex projective variety and Zariski topology.