

A brief course in model categories

ABSTRACT.

In 1967, only three years after presenting his Ph.D. thesis on partial differential equations, Daniel Quillen published his seminal work “Homotopical Algebra”. It introduced the notion of a “model category”: a framework that makes it possible to study homotopy theory in a setting that might be very different from the world of topological spaces. The use of model categories has led to a small revolution in homotopy theory; its benefits comprise better models for stable homotopy theory, the development of higher categories and the proof of the Milnor conjecture in algebraic geometry by Voevodsky.

After providing the historical background, I will recall some important definitions and facts from the classical homotopy theory of spaces. These will lead us to the much more general notion of a model category and its “homotopy theory”. We will discuss the most important examples, including the standard model structure for spaces and a nice model structure for chain complexes of R -modules. In the end, we will come up with the associated “homotopy categories” and learn how to compare different model structures.

Duration: approx. 75 minutes. My presentation will be as self-contained as possible; the audience should be familiar with the notions of a category and a functor as well as some of the basics from topology, though.