AN APPLICATION OF HOMOTOPY THEORY TO 3-MANIFOLDS

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In this talk I would like to promote the language and techniques of homotopy theory and basic algebraic topology in the study of (smooth) manifolds. To illustrate the connection between these areas of mathematics I will prove the following

Theorem *Let M be a smooth, closed, orientable 3-manifold. Then the tangent bundle TM is trivial, i.e., M is* parallelizable*,*

using methods only from algebraic topology and homotopy theory.

If time permits I want to reverse the point of view and show how understanding manifolds (in particular their characteristic classes) can help to answer questions in classical algebraic topology or homotopy theory, as for example the following

Theorem *In the cohomology $H^*(BU(2); \mathbb{F}_2)$ we have the relation*

$$\mathrm{Sq}^2(c_2) = c_1 \cdot c_2.$$

But, if at all, I will only briefly talk about the second theorem. Nevertheless of course, I will give a reason why this relation is interesting.